# Math 408C Practice Final

Evaluate the following limits.

1. $\lim_{x\to 2} \dfrac{x^2-4}{x^3-8}$

2. $\lim_{x\to 0} \dfrac{\sin 3x}{\tan 5x}$

3. $\lim_{x\to -\infty} \dfrac{\sqrt{2+3x^2}}{1-5x}$

4. $\lim_{x\to \infty} \sqrt{x^2+x} - \sqrt{x^2+10}$

Find $dy/dx$.

5. $y = \dfrac{x}{\sqrt{x^2-4}}$

6. $\sqrt{x} + \sqrt{y} = \sqrt{5}$.

7. $y = \ln\left(\dfrac{x^4}{1+x^3}\right) + 7^{x^{2/3}}$

8. $y = x^{1/x}$

9. $\ln \dfrac{\sec x + \tan x}{\sec x - \tan x}$

10. $y = e^{\tan^{-1} x}$.

Evaluate the integrals.

11. $\displaystyle\int \frac{x^3+1}{x^2}\, dx$

12. $\displaystyle\int \frac{\cos x}{\sqrt{\sin x}}\, dx$

13. $\displaystyle\int x^3 \tan^{-1} 3x\, dx$

14. $\displaystyle\int_0^2 \frac{x\, dx}{x^2+2}$

15. $\displaystyle\int \frac{3}{(x-4)(x^2+1)}\,dx$

16. $\displaystyle\int \sqrt{5-x^2}\,dx$

17. $\displaystyle\int \frac{10x^8}{x^6-1}\,dx$

18. $\displaystyle\int \tan^3 x \sec^6 x\,dx$

Graph the following, showing all important features.

19. $y = x^4 - 32x + 48$

20. $y = \dfrac{x^2}{x^2-1}$

21. $y = x^2 + \dfrac{2}{x}$

22. $y = \dfrac{\sin x}{1+\cos x}$

23. $y = e^{-x}\sin 2x$

24. Find the volume generated when the area bounded by $x = \sec y$, $x = 0$, $y = 0$ and $y = \pi/3$ is rotated about the $y$-axis.

25. The volume of a sphere is decreasing a the rate of $12\pi$ cubic feet per minute. Find the rates at which the radius and surface area are changing at the instant when the radius is 20 feet.

26. Find that number which most exceeds its square.

27. Find the points on the curve $x^2 - y^2 = 1$ which are nearest the point $(\sqrt{2}, 0)$.

28. A round hole of radius $\sqrt{3}$ ft is bored through the center of a solid sphere of radius 2 ft. Find the volume cut out.