

# M427K Final Exam

Name \_\_\_\_\_

NO NOTES. NO CALCULATORS.

1. Find the general solution of  $y' - 2xy = x$ .
2. Use the method of undetermined coefficients to find the general solution of  $y'' - y' - 2y = 4x^2$ .
3. The function  $y = 1$  is a solution to  $ty'' - 2y' = 0$ . Find another solution and show it's linearly independent to the first solution.
4. Let  $u(x, t)$  be a function of two variables. If possible, use the method of separation of variables to write  $u_{xx} + u_{tt} + 5u = 0$  as two different ordinary differential equations.
5. Use Laplace transforms to solve the IVP:  $y' + y = \sin(x), y(0) = 1$ .
6. Find the eigenvalues and eigenfunctions of the BVP  $y'' + \lambda y = 0, y(0) = 0, y'(0) = 0$ .
7. Find the recursion formula for the power series solution to  $y'' - xy' + 2y = 0$  about the  $x = 0$ .
8. The function  $f(x) = 1 + (1/2)x$  on the interval  $0 \leq x \leq 2$ . Find the Fourier sine series of this function.
9. Let  $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ . Solve  $\mathbf{x}' = A\mathbf{x}, \mathbf{x}(0) = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ .
10. Determine whether  $x = 0$  is a regular singular point of  $x^3y'' + 2x^2y' + y = 0$ .
11. (Ultramegabonus) Solve the following BVP involving the *wave* equation, where  $u(x, t)$  is the position of a string with fixed ends at time  $t$ .  
 $4u_{xx} = u_{tt}, u(0, t) = 0 = u(6, t), u(x, 0) = f(x)$ , where  $f(x) = x$  on the interval  $0 \leq x \leq 6$ .