

M427K Midterm Exam

Name _____

NO NOTES. NO CALCULATORS.

1. Find the general solution of $(y - x^3) dx + (x + y^3) dy = 0$.
2. Find the general solution of $y' + \frac{2t}{1+t^2}y = \frac{\cot t}{1+t^2}$.
3. Find the general solution of $y'' + 16y = 3te^{2t}$.
4. Find the general solution of $y'' - 2y' + y = \frac{e^t}{t^2 + 1}$.
5. Note that $y_1 = e^x$ is a solution to $xy'' - (2x + 1)y' + (x + 1)y = 0$. Solve the IVP consisting of this equation and the initial conditions $y(1) = 1, y'(1) = 1$. Also find the largest interval over which the solution is valid.
6. For the autonomous equation $y' = 2(y^2 - 1)$, determine the equilibrium solutions, classify their stability, and sketch a few representative solutions.
7. Using $\phi_0 = 0$, find the next two Picard iterates for the IVP $y' = -2t(y - 1), y(0) = 0$.
8. Find the first three nonzero terms of each of two linearly independent solutions to $(2 + x^2)y'' - xy' + 4y = 0$ about $x_0 = 0$.
9. A 1000-gallon tank initially contains 200 lbs. of salt dissolved in 600 gallons of water. A brine solution containing 2lbs. of salt per gallon is pumped into the tank at a rate of 3 gal./min. The well-mixed solution is pumped out at a rate of 1 gal./min. How much salt is in the tank when it is full?
10. Express the general solution of $t^2y'' + ty' + 4y = 0$ as a linear combination of two *real-valued* functions, and show that they are linearly independent.