M427L Summer 2008 Final Exam Name_____

NO NOTES. NO CALCULATORS.

1. Rewrite $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$ with order of integration $dx\,dy\,dz$.

2. Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^2\,dz\,dy\,dx$.

3. Tell whether $\mathbf{F}(x,y,z) = (2xz+\sin y)\vec{\boldsymbol{\imath}} + x\cos y\vec{\boldsymbol{\jmath}} + x^2\vec{\boldsymbol{k}}$ is conservative. If so, find a potential function for $\mathbf{F}$.

4. Evaluate $\int_{0}^{1}\int_{2y}^{2} 4\cos(x^2)\,dx\,dy$.

5. Use Lagrange multipliers to find the minimum value of $f(x,y,z) = x^2 + y + z^2$ subject to the constraint $2x + y + 4z = 6$.

6. Find the curvature of $\mathbf{c}(t) = \langle t, t^2, 1-t^2\rangle$ at the point where $t = 1$.

7. Let $\mathcal{C}$ be the curve given by $\vec{\boldsymbol{r}}(t) = \langle t, t^2, t^3\rangle$, $0 \le t \le 1$, and let $\mathbf{F}(x,y,z) = xy\vec{\boldsymbol{\imath}} + yz\vec{\boldsymbol{\jmath}} + zx\vec{\boldsymbol{k}}$. Evaluate $\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{s}$.

8. Evaluate $\iint_{\mathcal{S}} yz\,dS$, where $\mathcal{S}$ is the part of the plane $x + y + z = 1$ in the 1st Octant.

9. Evaluate $\oint_{\mathcal{C}} xy\,dx + x^2y^3\,dy$ where $\mathcal{C}$ is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$ oriented counter-clockwise.

10. Let E be the region bounded by $z = 1 - x^2$, $z = 0$, $y = 0$, and $y + z = 2$. Let $\mathcal{S}$ be the surface of E. Express the flux of the vector field $\mathbf{F}(x,y,z) = xy\vec{\boldsymbol{\imath}} + (y^2 + e^{xz^2})\vec{\boldsymbol{\jmath}} + \sin(xy)\vec{\boldsymbol{k}}$ over $\mathcal{S}$ as a triple integral.

11. Let $\mathbf{F}(x,y,z) = \langle yz, xz, xy\rangle$ and $\mathcal{S}$ be the part of the surface $z = 9 - x^2 - y^2$ that lies above $z = 5$, oriented upward. Use Stokes' Theorem to evaluate $\iint_{\mathcal{S}} (\nabla \times \mathbf{F}) \bullet d\mathbf{S}$.

12. Find the area of the cap cut from the sphere $x^2 + y^2 + z^2 = 2$ by the cone $z = \sqrt{x^2 + y^2}$.

13. Evaluate $\int_{\mathcal{C}} x^2\,dx + yz\,dy + (y^2/2)\,dz$ along the line segment C joining $(0,0,0)$ and $(0,3,4)$.

14. Find the area inside the loop of the curve $x = t^2 - 3$, $y = (t^3/3) - t$.

15. Let C be the curve of the intersection of $(x-1)^2 + 4y^2 = 16$ and $2x + y + z = 3$, oriented counter-clockwise when viewed from above. Let $\mathbf{F}(x,y,x) = (z^2 + y^2 + \sin x^2)\mathbf{i} + (2xy + z)\mathbf{j} + (xz + 2yz)\mathbf{k}$. Evaluate $\oint_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{s}$.

16. Find a parametrization of the surface $x^3 + 3xy + z^2 = 2$.