# M427L Midterm Exam                                                  Name________________

NO NOTES. NO CALCULATORS.

1. Find and classify the critical points of $f(x, y, z) = x^2 + y^2 + z^2 + yz + xz$.

2. Find the length of the curve $\mathbf{c}(t) = (t \cos t, t \sin t, \frac{2\sqrt{2}}{3} t^{3/2}), 0 \leq t \leq \pi$.

3. Let $\mathbf{F}(x, y, z) = (x^2 - y, 4z, x^2)$. Find div $\mathbf{F}$ and curl $\mathbf{F}$.

4. Express the equation $x^2 + y^2 = 1$ in spherical coordinates.

5. Find the point where the lines $\mathbf{c}_1(t) = (1 - t, 2t, -3 + 2t)$ and $\mathbf{c}_2(t) = (2t + 2, 3t + 19, 4t + 19)$ intersect. Then find the equation of the plain containing both lines.

6. Let $f(x, y) = \sqrt{xy + 1}$. Write the derivative and Hessian of $f$ at the point $(2, 4)$. Then use a quadratic approximation to estimate $f(2.03, 3.99)$.

7. Suppose $u = f(x, y)$ and $x = r \cos \theta$ and $y = r \sin \theta$. Show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

(This is a chain rule problem.)

8. Let $f(x, y) = 3x^2 - y^2$. Find all unit vectors $\mathbf{v}$ such that $D_{\mathbf{v}} f(1, 2) = 0$.

9. Let $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be a $C^2$ vector field. Show that $\nabla \bullet (\nabla \times \mathbf{F}) = 0$.

10. Find $\mathbf{T}, \mathbf{N}, \mathbf{B}, \kappa$, and $\tau$ for the curve $\mathbf{c}(t) = (3 \sin t, 4t, 3 \cos t)$ at the point where $t = \pi/2$. Sketch the curve and show these quantities on your sketch.