M408D Prac. Test 1

1. Find $\lim_{n\to\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$.

2. Use your knowledge of p-series and geometric series to determine all values of q for which each of the following series converge:

a. $\sum_{k=0}^{\infty} \frac{\pi^k}{q^{2k}}$ b. $\sum_{k=1}^{\infty} \left(\frac{1}{k^q}\right)^3$ c. $\sum_{k=2}^{\infty} \frac{1}{\ln q^k}$ d. $\sum_{k=2}^{\infty} \frac{1}{(\ln q)^k}$

3. Tell whether the following series converge:

a. $\sum_{k=1}^{\infty} \frac{2k-1}{3k^3-k}$ b. $\sum_{k=0}^{\infty} \frac{k^6 3^k}{(k+1)!}$ c. $\sum_{k=1}^{\infty} \ln\left(1+\frac{1}{k}\right)$

4. Evaluate $\int_0^{\infty} \frac{4}{x^2+9}\,dx$.

5. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{1}{\sin x}\,dx$.

6. Evaluate $\lim_{n\to\infty} \left(1+\frac{3}{n^2}\right)^n$.

7. Evaluate $\lim_{x\to\infty} (x+1)\sin\frac{1}{x+1}$.

8. Tell whether the sequence $a_n = \frac{\ln(n^2+1)}{n}$ converges. If so, find its value.

In problems 9 through 13 tell whether and why the series converges:

9. $\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1}$

10. $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$

11. $\sum_{n=1}^{\infty} \frac{n+\cos n}{n^3+1}$

12. $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$

13. $\sum_{n=1}^{\infty} \frac{e^{2n}}{(2n-1)!}$

14. Evaluate $\lim_{x\to 0} \frac{\sin x - \tan x}{x^3}$.

15. Find the radii and intervals of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}$ and $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$.